A short treatise on robots' kinematic geometry and kinetics.

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Freedoms, Constraints, and Mobility.

Motion of linkages: Screws and spatial motions.

Freedom and Mobility: Freedoms, unfreedoms, connectivity, mobility;

Grübler-Kutzbach's mobility criterion and examples;

Type-, size- and number-syntheses.

Degrees of Freedom and Kinematic Structure

Definition (Connection Degree)

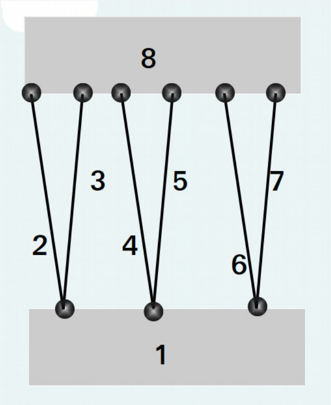
For any manipulator joint, we shall mean its connection degree to be the number of links attached it.

Quiz

What is the connection degree of the u-joints of a Stewart-Gough platform.

Members and Dual Graphs

Dual graph of a Stewart platform



Degrees of Freedom and Structure

Members and Freedoms

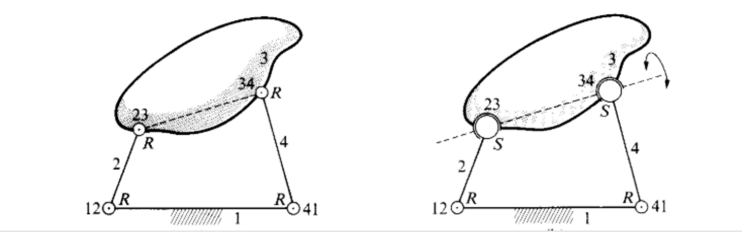
Degrees of freedoms (or freedoms) concerns the relative motion of members of a pair that do not touch one another directly.

Connectivity

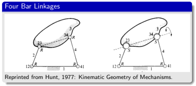
By the dual graph of the Stewart platform as seen on Frame 5, the total number of freedoms that connect the two members (1 and 8) that do not connect to one another directly is six.

Planar Linkages

Four Bar Linkages



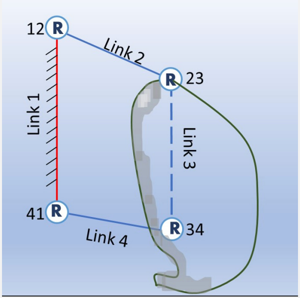
Reprinted from Hunt, 1977: Kinematic Geometry of Mechanisms.



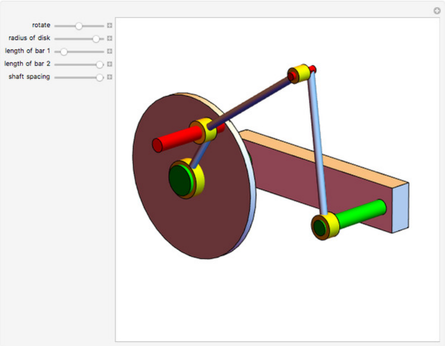
The planar $RRRR$ linkage, (*left*) is modified in (*right*) to an $RSSR$ linkage to allow spatial spin-movement of the coupler 3; the connectivity $\mathscr{C}_{13} = 2$.

Freedom from Connectivity

A (Hacked) Four-Bar Linkage



A Four Bar Linkage



Courtesy of Sándor Kabai, Wolfram Demonstrations Project, October 2007.

Couplings and Freedom

Couplings and Freedom

Links $2\&4$ complete a coupling or connection between links $1\&3$.

Connectivity

The R-pairs are said to have a connectivity of $\mathscr{C}_{ij} = 1$ for all $i, j = 1, 2, 3, 4$. Thus, total degree of freedom is 1.

Mobility of Mechanisms

The Mobility and Relative Mobility, $\mathfrak{M}$

Simply put, the number of a mechanism's freedoms is its mobility, or relative mobility, $\mathfrak{M}$.

The Mobility, $\mathfrak{M}$

It specifies the independent variables needed to determine every relative location of a mechanism's members with respect to one another.

A Note on Serial and Parallel Mobility

A little tricky to determine for parallel mechanisms but straightforward for serial mechanisms.

Mobility of Mechanisms

Quiz

What is the mobility $\mathfrak{M}$ of the $RSSR$ four bar linkage of Frame 7? Why?

Quiz

What is the mobility $\mathfrak{M}$ of the $RRRR$ four bar linkage of Frame 8? Why?

Definition (The mobility criterion (well, not yet))

Let's not get ahead of ourselves. A little introduction to screws are in order for us to grasp the Grübler-Kutzbach mobility criterion.

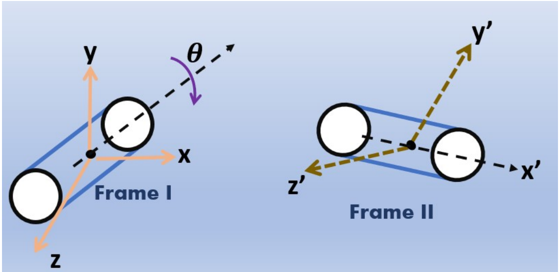
Inhomogeneity of Displacements and Angles

Quiz: Three translations and three rotations are ill-posed for uniquely determining the freedoms of a body. Why?

They are not homogeneous.

For true kinematic wholeness and generality, displacement that is purely translatory and purely rotary is needed.

Need for Unique Representations



There exists infinite possible ways of movement by which the point on the rigid body in Frame I can be effected to be transferred to the location of the point on Frame II and vice versa.

Screws for Kinematic Generality

Need for Screws

From a kinematic standpoint, six homogeneous screw coordinates – each having an independent screw freedom – are needed to uniquely determine a rigid body's location.

Definition (What is a screw anyway?)

A screw is a straight line in space, called the axis, with an associated direction, called pitch, p.

Screws in Mechanics: A History Snippet

Screws Through Time

Michel Chasles(1793 – 1880): Rigid Body Displacements (Kinematics).

Louis Poinsot (1777 – 1859): Geometrical Mechanics (Kinetics).

Sir Robert Stawell Ball, F.R.S, LL.D. (1840-1913): Irish Astronomer who popularized screw theory in his day.

Screws in Mechanics: A History Snippet

Screws Through Time

The Death of Screw Theory

After World War I, interest in Screw theory declined. Several possible reasons for this.

- Ball died in 1913, he had no students.
- Other British/Irish mathematicians who might have carried these ideas forward died young,
  - Clifford, a contemporary and friend of Ball's died 1879 aged 33.
  - Charles Jasper Joly (1864-1906) studied under Helmholz and Königs in Berlin, was a successor of Ball's as Royal Astronomer of Ireland.
  - Arthur Buchheim (1859–1888), studied under Klein at Leipzig. Taught at Manchester grammar school, died aged 29.
- Relativity became popular, and Euclidean geometry less so. Ball joked, "The Theory of Screws is now all done; it is quite obsolete; it is all going over into non-Euclidean space."

Kept alive in Soviet Union, Kotelnikov and others.

Rediscovered by Mechanical Engineers

In the 1960s two mechanical engineers in Australia rediscovered Balls work. Ball's theory of screws was just what they needed to study mechanisms.

- Kenneth Henderson Hunt (1920–2002) was born in the UK, worked at Monash University. "Kinematic Geometry of Mechanisms", first published in 1978. Applied screw theory to the problem of designing constant velocity joints.
- Jack Raymond Phillips (1923–2009) University of Sydney. Studied agricultural machinery, (trailed disc ploughs) and the mechanics of the lobster's claw. Two volume work "Freedom in Machinery: Introducing Screw Theory".

Courtesy of J.M. Selig, IROS 2018 Screw Theory Tutorial.

Screws for Kinematic Generality

Simplest (Unique) Representation of Displacements

Chasles(1793 – 1880) showed that any given displacement of a rigid body can be uniquely represented as the rotation of the body about an axis (the screw axis) followed by a translation parallel to that axis (the pitch).

Michel Chasles and Screws

Chasles called this unique transformation screws. Chasles is responsible for the Euclidean description of the motion of a rigid body in space and he made lasting contributions to theories of rigid body dynamics.

Screws for Kinetic Generality

Simplest (Unique) Representation of Forces

Poinsot(1777 – 1859) showed that any system of forces acting on a rigid body can be represented by a single force, together with a couple acting along the normal to the force in a plane.

Louis Poinsot and Geometrical Mechanics

"Everyone makes for himself a clear idea of the motion of a point, that is to say, of the motion of a corpuscle which one supposes to be infinitely small, and which one reduces by thought in some way to a mathematical point." ∼ Louis Poinsot, 1834.

Freedoms, Unfreedoms, and Mobility

Freedom and Constraints

Suppose a screw $\boldsymbol{f} = (f_1, \cdots, f_6)$ “fixes” a body in 3D space.

- Each constraint $u_i \neq f_j$ for $(i, j) \in \{1, \cdots, 6\}$.
- Rather each u_i has influence on every $\{f_i\}_{i=1}^{6}$.
- Each u_i from the six independent equations, $g(s_1, s_2, s_3, s_4, s_5, s_6) = 0$, suppresses a freedom, f_i.
- Progressively relaxing each u_i, or unfreedom, adds an extra body f_i.

Freedoms, Unfreedoms, and Mobility

Freedom and Unfreedoms

Suppose the total freedoms is $\boldsymbol{f}$ and the total unfreedoms is $\boldsymbol{u}$, then

$$\boldsymbol{u} + \boldsymbol{f} = 6.$$

Note: A rigid body's freedoms is also referred to the dimension of its configuration space.

Relative Freedoms

Suppose there are a total of n unconstrained bodies. Suppose further that we choose one out of the bodies as a reference body. Then the total number of relative freedoms is $6(n-1)$.

Freedoms, Unfreedoms, and Mobility

Constraints and Joints

Now, consider k independent constraints[a] such as joints along points, lines, curves or surfaces.

[a]NB: The total *allowable* constraints is 5 for a body in relative motion. 6 for a fully rigid body.

The Mobility Criterion

Let the constraint of joint, i (e.g. a joint along points, lines, curves or surfaces) be u_i. Then the mobility criterion $\mathfrak{M}$ is

$$\mathfrak{M} = 6(n-1) - \sum_{i=1}^{k} u_i. \tag{1}$$

General Grübler-Kutzbach Mobility Criterion

General Grübler-Kutzbach Mobility Criterion

Recall that $\sum_i u_i + f_i = 6$ from Frame (21) so that

$$\mathfrak{M} = 6(n - k - 1) - \sum_{i=1}^{f} f_i. \tag{2}$$

Exceptions: Relative Planar and Spherical Motions

For bodies restricted to relative planar or spherical motions, the total freedoms + constraints is 3 (not 6)!

$$\mathfrak{M} = 3(n - k - 1) - \sum_{i=1}^{f} f_i. \tag{3}$$

General Grübler-Kutzbach Criterion References

The Grübler-Kutzbach Mobility Criterion References

Attributed to Grübler:

Schoenflies, Arthur, and M. Grübler. "Kinematik." In Encyklopädie der Mathematischen Wissenschaften mit Einschluss ihrer Anwendungen, pp. 190-278. Vieweg+Teubner Verlag, Wiesbaden, 1908;

Grübler, Martin Fürchtegott. Getriebelehre: eine Theorie des Zwanglaufes und der ebenen Mechanismen. Springer, 1917.

and Kutzbach:

Kutzbach, Karl. "Mechanische leitungsverzweigung, ihre gesetze und anwendungen." Maschinenbau 8, no. 21 (1929): 710-716.

Loops

Loops

A kinematic chain often comprises members called loops.

Binary Link

Members in a binary link constitute a single loop. Example: The four-bar linkage.

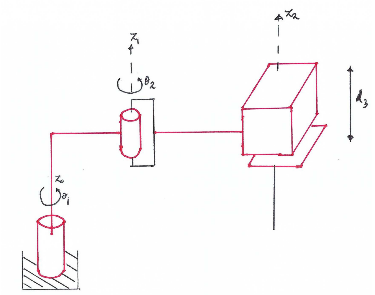
Single loops

For single loops, $k = n$ so that $\mathfrak{M} = \sum_{i=1}^{f} f_i - 6$.

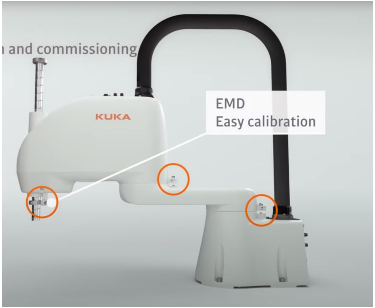
Mobility of Mechanisms

$\mathfrak{M} \leq 1$ for at least one actuator-pair to produce mobility at a successor joint which depends on that actuator-pair's input.

Mobility of Common Robot Configurations



Configuration of the SCARA Arm.



Courtesy of Fanuc America Inc.

Mobility of The SCARA Robot

Mobility Analysis

Two rotary joints. One prismatic joint acting along the z axis, and constrained along the xy plane.

Mobility Parameters

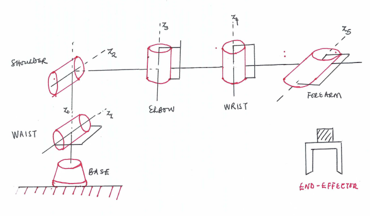
Four rigid bodies (links). Three constraints. Four freedoms. Therefore, $\mathfrak{M} = 6(4 - 3 - 1) + 4 = 4$

Mobility Analysis of The Universal Robot

©Universal Robots A/S, DK.

The Revolute Arm

Falls under so-called RRR kinematic arrangements. Also called a revolute, elbow, or anthorpomorphic manipulator.

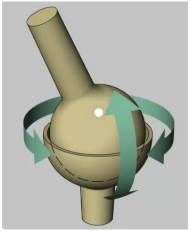


$n = 6$; $k = 5$; $f = 6$

$\therefore \mathfrak{M} = 6(n - k - 1) + \sum f_i$

$\implies 6(6 - 5 - 1) + 6$ or $\mathfrak{M} = 6$.

Mobility of The Stewart-Gough Platform



Mobility of The Stewart-Gough Platform

Unconstrained bodies, n

There are six universal joints that connect the base platform to the prismatic linear actuators. There are six spherical joints that connect the top platform to the top of the prismatic actuators. Altogether, there are $n = 6 + 6 + 2$ or 14 unconstrained rigid bodies.

Constraints, k

Six u-joints. Six spherical joints. Six prismatic joints. Altogether, there are $f = 6 + 6 + 6 := 18$ constraints.

Freedoms, f

Each u-joints has two freedoms. Each spherical joint has three (rotary) freedoms. Each prismatic joint has one freedom. Altogether, there are $f = 6 \times 2 + 6 \times 3 + 6 \times 1 := 36$ freedoms.

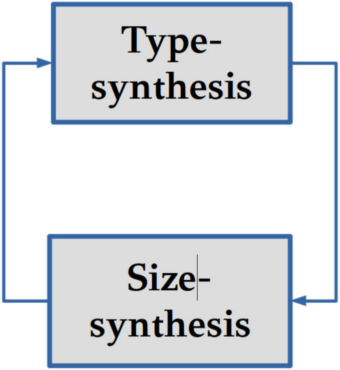
Mechanism Synthesis

Definition (Type-Synthesis)

What type of mechanism is appropriate for a task: A linkage or profile mechanism?

Definition (Size-Synthesis)

What major dimensions of the mechanism is to be synthesized?



Number Synthesis

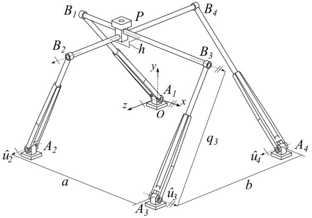
Definition (Number-Synthesis)

That which deals with the freedoms and constraints after the type- and size-synthesis of a mechanism, as well as a kinematic chain's structural analysis is termed the number-synthesis of the mechanism.?

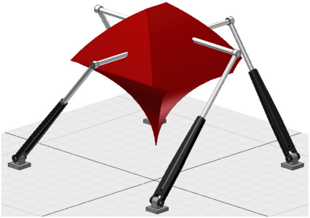
Quiz: Mobility of a Planar Parallel Mechanism

Quiz

Analyze the mobility of the mechanism below.



A planar parallel mechanism.
Reprinted from Garcia-Murillo et al.



Workspace of the mechanism.

Quiz: Mechanism Hints

Hint – Mechanism Description

Point $P = (P_x, P_y, P_z)$ is the interconnecting point for all the chains on the mobile platform and the top rods.

Hint – Mechanism Description

The rods that connects points B_1 and B_3, and points B_2 and B_4 are perpendicular. Both rods are connected to the moving platform by prismatic joints, which are separated from each other by a vertical offset h. $\hat{u}_i$ signifies universal joints.