A short treatise on robots' kinematic geometry and kinetics.

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Standard Texts – Modeling and Control

Robot Modeling and Control

Spong, Mark W., Seth Hutchinson, and Mathukumalli Vidyasagar. Robot modeling and control. Vol. 3. New York: Wiley, 2006.

Mathematical Modeling of Robots

Murray, R. M., Li, Z., & Sastry, S. S. (1994). A Mathematical Introduction to Robotic Manipulation. In Book (Vol. 29). https://doi.org/10.1.1.169.3957

Texts – Modeling, Control, and Mechanisms

Robot Modeling and Control

Lynch, K. M., & Park, F. C. (2017). Modern Robotics Mechanics, Planning, and Control.

Mechanisms' Kinematic Geometry

Hunt, Kenneth H., and Kenneth Henderson Hunt. Kinematic geometry of mechanisms. Vol. 7. Oxford University Press, USA, 1978.

Texts – Screws and Kinematics

Screw Theory

Ball, Robert Stawell. A Treatise on the Theory of Screws. Cambridge university press, 1998.

Mechanisms' Kinematic Geometry

Hunt, K. H. (2019). Structural Kinematics of In-Parallel-Actuated Robot-Arms. 105(December 1983), 705–712.

Lecture One Outline

Mechanism Components

Kinematic geometry. Mechanisms.

Joints: Joint closure; Pairs; Couplings.

Lower pairs and linkages; Higher and lower pairs.

Motions: Planar and spherical motions.

Synthesis: Type-, number-, and size-syntheses.

Preamble.

Mechanics

Mechanics is an indirect study of nature via bodies – essentially the mathematical abstractions of common natural things; the mass is an *allocation* in *place* to each body.

Geometry

Geometry, deals with the theory of places; geometry is the bedrock of robotics, control theory, and many fields of modern engineering and the physical sciences.

Mechanics Overview.

Definition (Motion)

When a place undergoes body transformation in the course of time, we have motion.

Preamble – Mass, Body, Rigid Body Motion.

Definition (Body – Truesdell, 1977.)

By a body, we shall mean the closure of an open set in some measure space Ω over which a non-negative measure M, called the mass, is defined, and that M can be extended to a Borel measure over the $\sigma-$ algebra of Borel sets in Ω.

Preamble – Mass, Body, Rigid Body Motion.

Bodies – Truesdell, 1977.

That in mechanics which deals with
(i) mass points, which occupy a single point at any one time;
(ii) rigid bodies, which never deform;
(iii) strings and rods and jets, which are 1-dimensional;
membranes and shells, that sweep out surfaces;
(iv) space-filling fluids and solids e.t.c. are termed bodies.

Statics, Dynamics, Rigid Body (Motion).

Statics and Dynamics

That which studies putative equilibria is referred to as statics. That which concerns motion of all sorts is referred to as dynamics. The dynamics that are specific to particular bodies are termed constitutive.

Statics, Dynamics, Rigid Body (Motion).

The Rigid Body

A rigid body does not stretch, buckle, contract, bend, twist, nor deform. Well, not really!

The Rigid Body

As engineers and roboticists, we judge kinematic rigid hardware with the expectation that kinematic changes do not depart from rigid-body predictions.

Statics, Dynamics, Rigid Body (Motion).

The Rigid Body

We expect that localized stresses, active noise, vibrations and heat e.t.c will not cause reasonable departures from expectations.

Rigid Body Motion

That motion that preserves distance between all points in a body is termed a rigid body motion.

Rigid Body Motion

At issue are components of a rigid body's movement w.r.t to a fixed or moving frame of reference. In its most basic form, this movement is parameterized by displacement (and is sometimes time-varying e.g. for a continuum body). When solving for the movements of bodies, it is often useful to include velocities (twists) in order to characterize the motion.

Kinematics vs. Kinetics

Dynamics

$$\dot{x} = f(t; x, u), \quad x(t_0) = t_0 \tag{1}$$

$$\dot{x} = f(t; x) + g(t; x, u), \quad x(t_0) = t_0 \tag{2}$$

Definition (Kinematics.)

Kinematics is the English version of the word *cinématique* coined by A.M. Ampère (1775-1836), who translated it from the Greek word *κίνημα*.

Kinematics vs. Kinetics.

Definition (Truesdell)

That part of a system's dynamics that involves its motion by displacement – both linear and angular – and separated from motions owing to forces and torques, together with the successive derivatives with respect to time of all such displacements (this includes velocities, accelerations, and hyper accelerations) all form the kinematics of a rigid, continuum or laminae of bodies.

Kinematics vs. Kinetics

Kinetics

The motion of bodies can also be conceived as resulting from the forces' action. Energy, temperature, and calory of a body are resultant effects of gains or loss of heat. Motions arising as a result of these are called kinetics.

Kinematics vs. Kinetics.

Definition (Kinetics – Technical Definition)

That part of a system's dynamics that involves its motion by forces, energy, torque, inertia, dynamic stability, and equilibrium and similar properties all form the kinetics of a rigid, continuum or laminae of bodies.

Kinematic Geometry.

Definition (Kinematic Geometry)

The solid geometry of relatively moving rigid bodies is termed the kinematic geometry of the rigid body. With motion, we'd have to include the successive derivatives of the displacement such as acceleration e.t.c as the 'laws of motion' stipulates in mechanics.

Joints and Links

Links

Links may be rigid mechanical parts, elastic, (vulcanized) rubber components, diaphragms, conveyor belts, spring-damper systems e.t.c.

An Elementary Joint or Kinematic Pair.

An elementary joint or a kinematic pair consists of touching two links together at one point – then ensuring a single contact point is continuously maintained throughout relative movement.

Joint (Contact) Kinematics

Contact Kinematics

A body may slide or slip across a plane or surface, or roll over another body.

Joints

Joints are the result of the connecting points between two or more rigid bodies.

Definition of a Mechanism

Definition (Author's Definition)

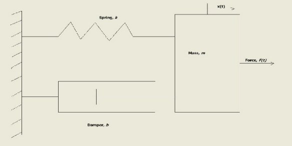
A connection of mechanical, magnetic, electrical, hydraulic, or pneumatic components forming an assemblage, meant for moving rigid, semi-rigid or non-rigid bodies via a controlled generation of (sometimes constrained) motion.

Kenneth Hunt (1978)

A means of transmitting, controlling, or constraining the relative movement between parts. Whenever we have an higher pair or more, we have a mechanism.

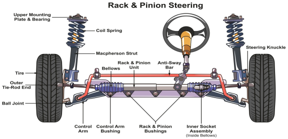
Mechanism Examples

Spring-Mass-Damper System

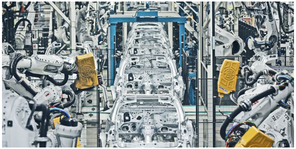


Excavator

Car suspension



Daimler Plant



Lower Pairs, Higher Pairs, Linkages

Lower and Higher Pairs

When elements of pairs touch one another over a substantial region of a surface covering a line, curve-surface, or point of contact, we have lower pairs. When they touch along a discrete line, curve-surface, or point of contact, we have higher pairs.

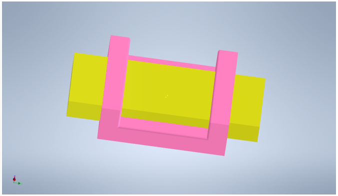
Linkage (Hunt, 1978)

If all joints of a mechanism or mechanical movement belong to lower pairs, we have a linkage.

Prismatic Pairs or P-pairs

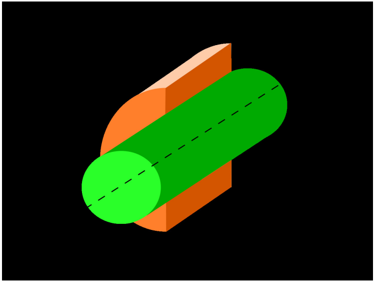
Hunt, 1978

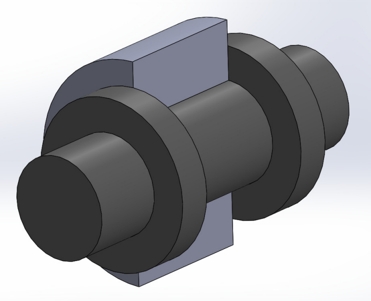
Formed by receding the axis of the revolution surface between two pairs to ∞ so that the curve that produces the surface moves parallel to itself, tracing a cylinder; or a polygonal-tracing curve generates a prism.



Revolute Pairs or R-pairs

One convex surface and one non-convex surface for a one degree of rotational freedom around the one joint the two surfaces make.





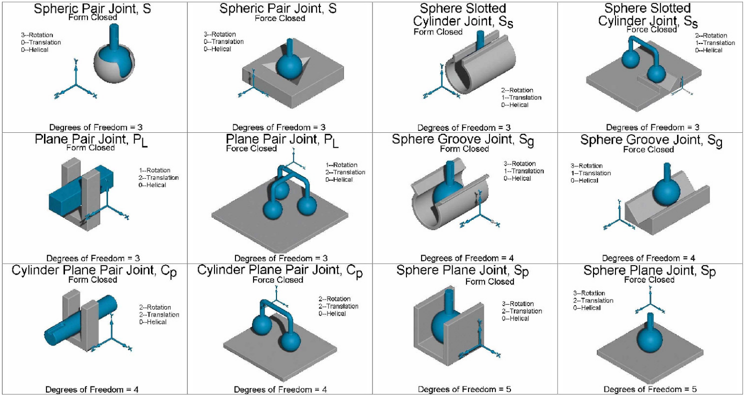
Revolute or Hinge or Turning or simply R-pairs with and without shoulder cutaway geometries. Credit: Wikimedia commons.

Helical- & U-Joints

Helical Joint

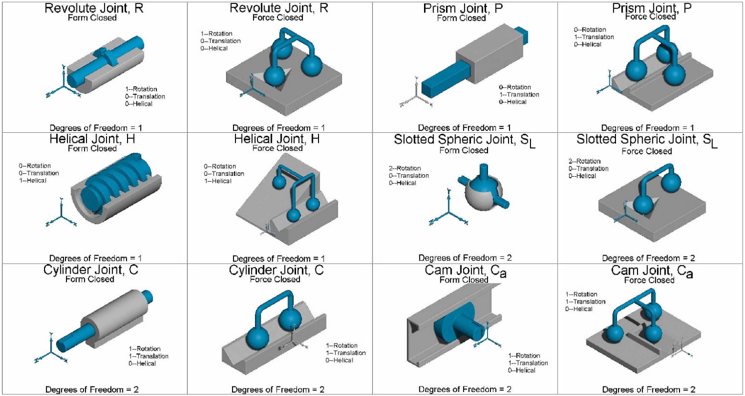
Universal Joint

Common Lower Kinematic Pairs



Credit: Wharton and Singh, 2001.

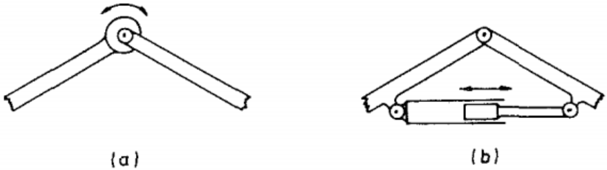
Common Lower Kinematic Pairs



Credit: Wharton and Singh, 2001.

Kinematic Geometry of Common Actuations

In-series vs. Parallel-actuated lower pairs



(a): In-series-actuated kinematic pair with a rotary joint that is actuated "about" the hinge. (b): Prismatic joint actuated "across" a hinge. Reprinted from Hunt, Kenneth. Structural Kinematics of In-Parallel-Actuated Robot Arms. Transactions of ASME. 1983.

Kinematic Chains

Kinematic Chains (Reuleaux, 1975)

We can explain the structural similarity of many mechanisms by parts of kinematic chains connected by pairs.

Kinematic chains

Kinematic chains are essentially the basic building structure of mechanisms . . . and robots!

Open Kinematic Chains

Chains

Open kinematic chains are based off the anthropomorphic construction of the human hand with cantilevered beam structures.

Chain Mechanisms and Error Amplification

Amplifies errors from waist (or base frame) all the way to the tool frame. Control difficult.

Control

Feedforward control: High power and precision hydraulic actuators for servo motors.
Sensory feedback control: Force sensing (Ernst, 1962).

Chains
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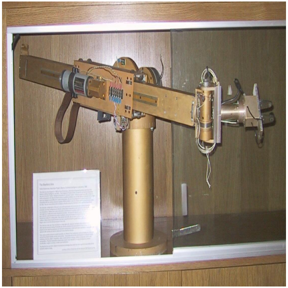
The PUMA arm is the world's first serial kinematic chain. Developer: Victor Scheinman, Stanford student in the '50's. Made several iterations. Patent Rights: Joe Engelberger, (Danbury Unimation, 1961). Joe – father of robotics – created world's first robotics company in '61.

Open Kinematic Lower Pairs

Definition (Ken Salisbury Jr., 1982)

"*[Robots are] our fascination with constructing mechanical analogues of ourselves... [this fascination] has led us to place all sorts of hopes and expectations in robot capabilities.*"

The PUMA Robot (1956).



The Stanford Arm (Infolab 1969).

Open Kinematic Chains

Open kinematic chains provide unstructured environmental interaction.

Project MAC, MIT.

Tomovic and Boni's pressure sensed grasp.

Binary robot vision system (McCarthy et al, 1963).

Open Kinematic Chains

Stanford Manipulator.

Boston arm.

The AMF (American Machines and Foundry) arm.

General electric's walking robot (1969).

Long Walk Towards Direct Drive Robot Arms

The 50's, 60's nd 70's witnessed use of hydraulics for (feedforward) position control.

For feedback control, force sensors and pressure sensors were used in closed-loop scenarios.

Electrical actuation meant that robots had to be operated at high speeds. Needs for gear reduction for safe operations at low speeds.

With gear reduction came backlash, friction, and associated expenses.

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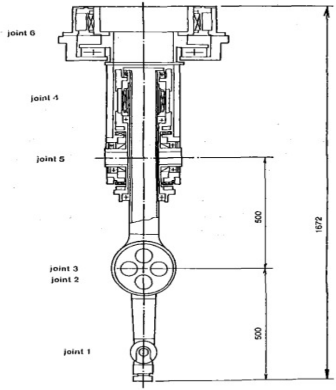
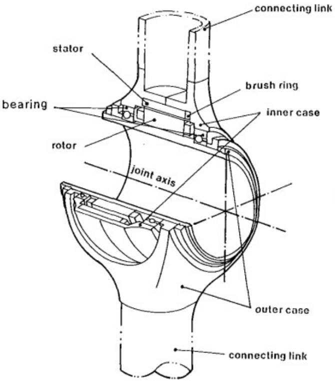
CMU DD I/II Arms: Workspace is donut shaped. OD: 90cm; ID: 21.7cm; $1.8m^2$ workspace area. Built by Harry Asada. Structural design similar to aircraft gimbal arm; Uses Samarium Cobalt rare earth magnet brushless DC motors on first 3 joints, and AlNiCo magnets on tip joints. No belts, transmissions making for faster transmitting of motions, less friction, low energy, low compliance. Each joint has complex AL housing which enables: (i) Control of geometrical relationships of bearing assembly; (ii) Control of servo components to bearing assembly; (iii) Controls of rotational axes to consecutive joints.

Direct Drive Robot Mechanism: CMU DD I Arm

Along came Harry Asada.

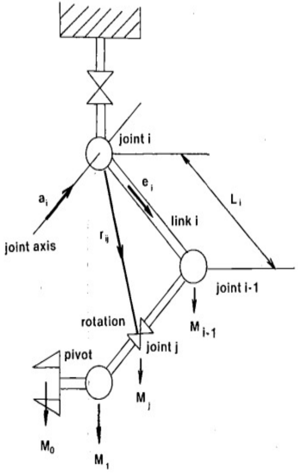


Arm Schematics Transmission

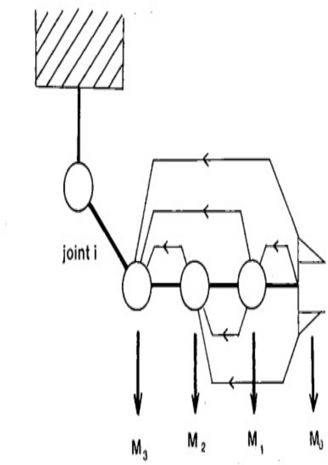


Joint schematic

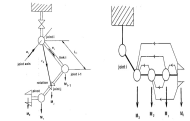
Direct Drive Robot Mechanism: CMU DD I Arm



Kinematic model



Errors Transmission

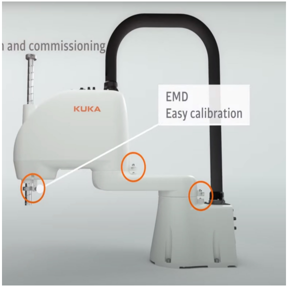


Kinematic model Errors Transmission

First direct-drive robot without a gearbox. Selective compliance in X-Y directions given its articulated jointed arms. One-freedom motion along Z direction given its constrained arm New generations such as Cobra i600/i800 include power amplifiers, system and servo controls etc embedded in the robot's base. Kuka Scara arm: Lightweight, fast, powerful, low maintenance, energy consumption, investment costs etc.

SCARA Robot Mechanisms

The Adept One SCARA robot (Debuted 1984).



Kuka's SCARA arm, 2022. ©Kuka Robotics

The Stäubli anthropomorphic arm.

Serial mechanisms research in the 80's

Mechanisms in the 80's

With the 80's came the arrival of PCs. Lots of research went into computational algorithms for the kinematics and kinetics of (mostly) anthropomorphic robot arms.

Active control schemes

Efficient recursive Lagrangian and computational methods for the gravitational and Coriolis forces in Newton-Euler equations.

Serial mechanisms research in the 80's

Feedback Linearization

Dynamics feedback linearization for precise bounds on manipulator performance.

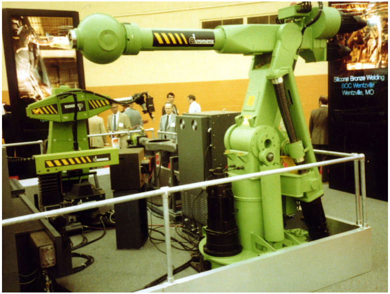
Automatix

Reconfigurable robots for various assembly ops.

Serial mechanisms research in the 90’s

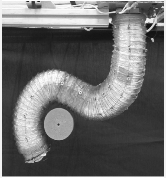
Robotworld

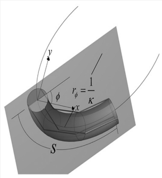
First industrial-scale reconfigurable robot and with machine vision components. RAIL scripting OS originally based on Motorola 68000, later on replaced by Apple Macintosh II.

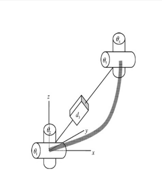


©Wikipedia

Hyper-redundant Continuum Robots

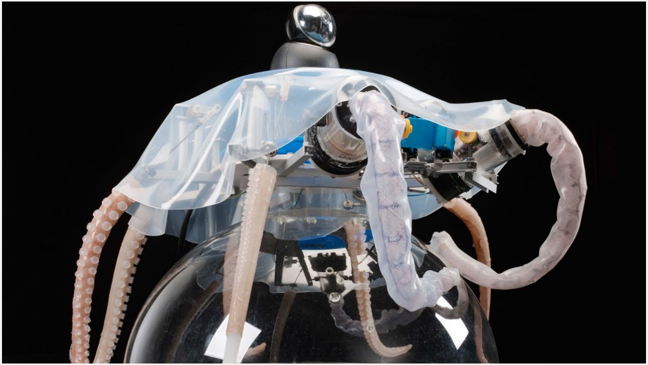






The elephant trunk continuum robot. Jones & Walker, T-RO 2006.
Inspiration: Muscular hydrostats in nature.

Hyper-redundant Kinematic Chains



An octopus-inspired soft robot. ©Cecilia Laschi.

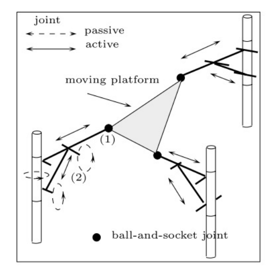
Parallel Robots

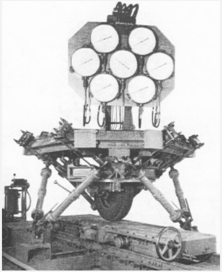
Mehlet 2015

A parallel robot is made up of an end-effector with n degrees of freedom, and of a fixed base, linked together by at least two independent kinematic chains. Actuation takes place through n simple actuators.

Parallel mechanisms: Stewart-Gough Platforms

Principles of a moving platform to test tyre wear and tear (Gough, 1947). Prototype, 1955.





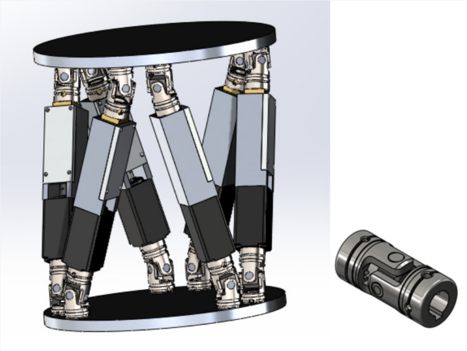
Left: Stewart's 1965 mechanism. *Right:* The original 1954 octahedral hexapod proposed by Gough. Courtesy: Parallemic.org.

Truss Robots

A multi-DOF Truss Robot. Courtesy of Penngineering (ICRA 2022, Philadelphia, PA).

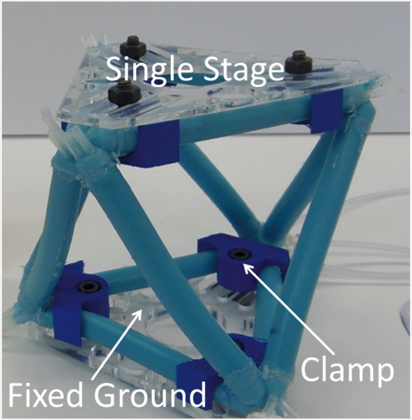
Closed kinematic chains

Connection degree ≥ 3.



A Stewart-Gough platform. SolidWorks Drawing Courtesy of Andrew Belcher. UChicago, 2018.

A Soft Stewart Platform



A soft 6-6 Stewart manipulator. Jonathan Hopkins, 2015.